Matrices Word Problems And Solutions

MASTERING MATRICES WORD PROBLEMS AND SOLUTIONS: A PRACTICAL GUIDE

If you have ever stared at a tangled set of equations from a real-world scenario and wondered how to untangle them efficiently, you are not alone. Many students and professionals encounter situations where multiple variables interact in ways that simple algebra struggles to capture. This is where matrices come into play. In this article, we explore **Matrices Word Problems And Solutions** in depth, providing clear explanations, step-by-step methods, and practical examples. By the end, you will not only understand how to set up matrix equations from word problems but also how to solve them confidently using techniques such as Gaussian elimination, matrix multiplication, and inverses. Whether you are preparing for an exam or applying linear algebra in data science, operations research, or economics, mastering these skills will transform the way you handle complex, interconnected data.

WHAT ARE MATRICES AND WHY USE THEM FOR WORD PROBLEMS?

A matrix is a rectangular array of numbers, symbols, or expressions arranged in rows and columns. In the context of word problems, matrices serve as compact representations of systems of linear equations. Instead of writing out each equation separately, you can express the entire system as a single matrix equation of the form **A · X = B**, where **A** holds the coefficients, **X** contains the variables, and **B** contains the constants. This representation is not only neat but also opens the door to powerful solution methods, such as using the inverse of a matrix or row reduction.

Word problems often involve multiple constraints and unknown quantities. Traditional substitution methods become tedious when there are three or more variables. Matrices provide a systematic, error-reducing approach that scales well to larger systems. Moreover, many real-world problems in engineering, economics, and computer science are naturally expressed in matrix form, making this skill highly transferable.

Key Matrix Operations You Need to Know

Before diving into specific word problems, let us briefly review the operations that will be used throughout the solutions:

- **Matrix addition and subtraction** – Adding or subtracting corresponding entries (only possible for matrices of the same dimensions).
- **Scalar multiplication** – Multiplying every entry of a matrix by a constant.
- **Matrix multiplication** – The product of an $m \times n$ matrix and an $n \times p$ matrix yields an $m \times p$ matrix. This operation is essential for representing linear transformations and solving systems.
- **Finding the inverse of a matrix** – For a square matrix A, if $\det(A) \neq 0$, its inverse A^{-1} exists. The solution to $A \cdot X = B$ is $X = A^{-1} \cdot B$.
- **Gaussian elimination (row reduction)** – Transforming an augmented matrix into row-echelon form to solve the system step by step.

TYPES OF MATRICES WORD PROBLEMS AND THEIR

SOLUTIONS

We will now examine several common categories of word problems that lend themselves to matrix methods. Each example includes the formulation, the step-by-step solution, and explanatory notes to help you understand the reasoning.

1. Solving Systems of Linear Equations

This is the most straightforward application. Many word problems describe relationships between several quantities that can be written as linear equations. For instance:

Example: A bakery sells three types of pastries: croissants, muffins, and scones. On Monday, they sold 20 croissants, 30 muffins, and 15 scones for a total revenue of $145. On Tuesday, they sold 25 croissants, 20 muffins, and 20 scones for $160. On Wednesday, they sold 30 croissants, 25 muffins, and 10 scones for $165. Find the price of each pastry.

Solution: Let x = price of a croissant, y = price of a muffin, z = price of a scone. The system is:

- $20x + 30y + 15z = 145$
- $25x + 20y + 20z = 160$
- $30x + 25y + 10z = 165$

Write this in matrix form: **A·X = B**, where

A = [[20,30,15],[25,20,20],[30,25,10]], **X** = $[x,y,z]^T$, **B** = $[145,160,165]^T$.

We can solve using the inverse matrix method or Gaussian elimination. Using an inverse (assuming non-singular): compute A^{-1} (by hand or with a calculator) and multiply by B. For educational purposes, let us perform Gaussian elimination on the augmented matrix:

Augmented matrix:
[20 30 15 | 145]
[25 20 20 | 160]
[30 25 10 | 165]

Step 1: Make the first pivot 1. Divide row1 by 20:
[1 1.5 0.75 | 7.25]
[25 20 20 | 160]
[30 25 10 | 165]

Step 2: Eliminate below: row2 ← row2 - 25·row1, row3 ← row3 - 30·row1:
[1 1.5 0.75 | 7.25]
[0 -17.5 1.25 | -21.25]
[0 -20 -12.5 | -52.5]

Step 3: Make second pivot 1: row2 ← row2 / (-17.5):
[1 1.5 0.75 | 7.25]
[0 1 -0.0714286 | 1.2142857]
[0 -20 -12.5 | -52.5]

Step 4: Eliminate above and below: row1 ← row1 - 1.5·row2, row3 ← row3 + 20·row2:
[1 0 0.8571429 | 5.4285714]
[0 1 -0.0714286 | 1.2142857]
[0 0 -13.928571 | -28.214286]

Step 5: Third pivot: row3 ← row3 / (-13.928571):
[1 0 0.8571429 | 5.4285714]

[0 1 -0.0714286 | 1.2142857]
[0 0 1 | 2.025] (approximately)

Step 6: Back-substitute: row1 ← row1 - 0.8571429·row3, row2 ← row2 + 0.0714286·row3:
[1 0 0 | 3.5]
[0 1 0 | 1.5]
[0 0 1 | 2.025]

Thus x = $3.50, y = $1.50, z = $2.025 (rounded). So croissants are $3.50, muffins $1.50, and scones approximately $2.03. This concise matrix solution avoids messy substitution.

2. Mixture and Blend Problems

Mixture problems often involve combining ingredients with different properties (price, concentration, etc.) to achieve a target. For example, a coffee blend uses three types of beans with different costs per pound. The word problem gives total weight and total cost constraints.

Example: A coffee shop blends three types of beans: Brazilian ($5/lb), Colombian ($6/lb), and Ethiopian ($8/lb). They want to produce 100 pounds of blend that costs $6.20 per pound. Also, the amount of Colombian beans must be twice the amount of Ethiopian beans. How many pounds of each should be used?

Solution: Let *b* = Brazilian, *c* = Colombian, *e* = Ethiopian. Equations:

- b + c + e = 100
- 5b + 6c + 8e = 620
- c = 2e → c - 2e = 0

In matrix form: A = [[1,1,1],[5,6,8],[0,1,-2]], B = $[100,620,0]^T$. Solve: using Gaussian elimination or inverse, we get b = 30, c = 40, e = 20. So 30 lb Brazilian, 40 lb Colombian, 20 lb Ethiopian. The matrix approach handles the extra constraint seamlessly.

3. Inventory and Supply Chain Problems

Retailers and manufacturers often track quantities of multiple items across several locations or time periods. Word problems may ask for unknown stock levels after transfers or sales.

Example: A company has three warehouses: A, B, and C. They store two products: widgets and gadgets. The inventory (in units) before a shipment is represented by the matrix **I** = [[150,200],[100,250],[180,120]] (rows: warehouses A,B,C; columns: widgets, gadgets). They receive a shipment that adds the matrix **S** = [[20,30],[10,15],[25,20]]. Later, they ship out matrix **O** = [[40,50],[30,40],[20,30]]. Find the final inventory matrix.

Solution: Final inventory = I + S - O. Compute element-wise:
For warehouse A: widgets = 150+20-40=130, gadgets=200+30-50=180.
B: widgets=100+10-30=80, gadgets=250+15-40=225.
C: widgets=180+25-20=185, gadgets=120+20-30=110.
Final matrix = [[130,180],[80,225],[185,110]]. This straightforward matrix addition/subtraction yields the answer.

4. Network Flow and Traffic Problems

Directed graphs (like traffic networks or electrical circuits) often give rise to systems of linear equations. The flow into a junction equals the flow out (Kirchhoff's current law).

Example: Consider a one-way street network with four intersections. The flows (cars per hour) on the streets are: between A and B: x_1, B and C: x_2, C and D: x_3, D and A: x_4, plus external inflows/outflows. Suppose at intersection A, inflow from outside is 100, outflow is

$x_1 + x_4 = 100$. At B, inflow $x_1 = 60 + x_2$. At C, inflow $x_2 = 30 + x_3$. At D, inflow $x_3 + 20 = x_4$. Find the flows x_1 - x_4.

Solution: Write the equations:

- $x_1 + x_4 = 100$
- $x_1 - x_2 = 60$
- $x_2 - x_3 = 30$
- $x_3 - x_4 = -20$

Matrix form: rows correspond to equations. Solve by Gaussian elimination. Augmented matrix:
[1 0 0 1 | 100]
[1 -1 0 0 | 60]
[0 1 -1 0 | 30]
[0 0 1 -1 | -20]

Row reduce: subtract row1 from row2 → row2 becomes [0 -1 0 -1 | -40]. Multiply row2 by -1: [0 1