Testing Statistical Hypotheses Worked Solutions

UNDERSTANDING THE FOUNDATIONS OF HYPOTHESIS TESTING

Statistical hypothesis testing is a cornerstone of data-driven decision making across scientific research, business analytics, and policy evaluation. At its heart, the process allows researchers to determine whether observed patterns in data are statistically meaningful or merely the result of random variation. However, mastering this discipline requires more than memorizing formulas; it demands a practical understanding of how to apply tests correctly and interpret results with confidence. This is where **testing statistical hypotheses worked solutions** become invaluable. By walking through complete examples from start to finish, learners can bridge the gap between abstract theory and real-world application.

Whether you are a student preparing for an examination, a data analyst validating a business hypothesis, or a researcher publishing findings, working through fully solved examples transforms hypothesis testing from a confusing set of rules into a logical, repeatable process. In this article, we will explore the fundamental structure of hypothesis testing, examine three detailed worked solutions, and highlight common mistakes to avoid when performing these essential statistical procedures.

WHAT IS STATISTICAL HYPOTHESIS TESTING?

Statistical hypothesis testing is a formal procedure for deciding whether to reject a default position, known as the null hypothesis, based on sample data. The null hypothesis typically represents no effect, no difference, or no relationship. Against this stands the alternative hypothesis, which represents the effect, difference, or relationship the researcher suspects is true.

The entire framework rests on probability theory. By calculating the likelihood of observing the obtained results assuming the null hypothesis is true, statisticians can quantify the strength of evidence against the null. If this probability, called the p-value, falls below a predetermined significance level, the null hypothesis is rejected in favor of the alternative.

To truly grasp this process, one must move beyond definitions and into the mechanics of actual calculations and decision-making. This is precisely why **testing statistical hypotheses worked solutions** are so effective as a learning tool. They show each step—from stating hypotheses to drawing conclusions—in a concrete, reproducible manner.

KEY COMPONENTS EVERY WORKED SOLUTION SHOULD INCLUDE

Before diving into specific examples, it is important to understand the essential elements that any complete hypothesis test must contain. A well-structured worked solution will always address these components:

- **Null and alternative hypotheses** stated clearly in both words and symbols.
- **Significance level** chosen before analyzing the data, typically 0.05 or 0.01.
- **Test statistic** calculation using the appropriate formula for the data type and research question.
- **Critical value or p-value** obtained from the relevant statistical distribution.
- **Decision rule** that compares the test statistic to the critical value or evaluates the p-value against the significance level.
- **Conclusion** stated in plain language that directly addresses the original research question.

When you study **testing statistical hypotheses worked solutions**, pay close attention to how each of these components flows logically into the next. This structure is not arbitrary; it ensures that the testing process remains objective and reproducible.

A STEP-BY-STEP FRAMEWORK FOR ANY HYPOTHESIS TEST

While there are many types of statistical tests, almost all follow a general framework. Mastering this framework makes it easier to approach any hypothesis testing problem methodically.

1. **State the hypotheses.** Define the null and alternative hypotheses based on the research question.
2. **Choose the significance level.** Select alpha before collecting or examining the data.
3. **Select the appropriate test.** Consider the type of data, sample size, and whether you are comparing means, proportions, or variances.
4. **Calculate the test statistic.** Use the correct formula for your chosen test.
5. **Determine the critical value or p-value.** Refer to the appropriate statistical table or use software.
6. **Apply the decision rule.** Reject the null hypothesis if the test statistic falls in the critical region or if the p-value is less than alpha.
7. **Interpret the result.** State the conclusion in the context of the original problem.

By repeatedly applying this structure through worked solutions, the process becomes second nature. Let us now examine three fully worked examples that demonstrate this framework in action.

WORKED SOLUTION 1: ONE-SAMPLE T-TEST

Scenario

A manufacturer claims that their energy drink contains exactly 80 milligrams of caffeine per serving. A consumer advocacy group tests 25 randomly selected servings and finds a sample mean of 76.8 mg with a sample standard deviation of 6.5 mg. At a significance level of 0.05, is there sufficient evidence to conclude that the mean caffeine content differs from the claimed 80 mg?

Step 1: State the Hypotheses

Null hypothesis: The true mean caffeine content is 80 mg. H_0: $\mu = 80$.
Alternative hypothesis: The true mean caffeine content is not 80 mg. H_1: $\mu \neq 80$.

This is a two-tailed test because we are checking for any difference, not a specific direction.

Step 2: Choose the Significance Level

Alpha = 0.05.

Step 3: Select the Test

We are comparing a sample mean to a known population mean, the population standard deviation is unknown, and the sample size is 25. The one-sample t-test is appropriate.

Step 4: Calculate the Test Statistic

The formula for the one-sample t-statistic is:
$t = (\bar{x} - \mu) / (s / \sqrt{n})$

Substituting the values:
$t = (76.8 - 80) / (6.5 / \sqrt{25})$
$t = (-3.2) / (6.5 / 5)$
$t = (-3.2) / 1.3$
$t \approx -2.4615$

Step 5: Determine the Critical Value

For a two-tailed test with alpha = 0.05 and degrees of freedom = n - 1 = 24, the critical t-values are ±2.064 (obtained from the t-distribution table).

Step 6: Apply the Decision Rule

The calculated t-statistic of -2.4615 falls below -2.064, which means it lies in the critical region. Therefore, we reject the null hypothesis.

Step 7: Conclusion

There is sufficient evidence at the 0.05 significance level to conclude that the true mean caffeine content is different from 80 mg. The consumer advocacy group has grounds to question the manufacturer's claim.

This **testing statistical hypotheses worked solution** shows the full path from raw data to a meaningful conclusion. Notice how each step builds logically on the previous one, and how the final conclusion directly answers the original question.

WORKED SOLUTION 2: TWO-SAMPLE T-TEST (INDEPENDENT SAMPLES)

Scenario

A company wants to compare the effectiveness of two training programs. Twelve employees are randomly assigned to Program A, and ten employees are assigned to Program B. After training, their productivity scores are measured. The mean score for Program A is 85.2 with a standard deviation of 4.8, while the mean for Program B is 80.1 with a standard deviation of 5.3. At alpha = 0.05, is there a significant difference in the mean productivity scores between the two programs? Assume equal variances.

Step 1: State the Hypotheses

H_0: $\mu_1 = \mu_2$ (the population means are equal)
H_1: $\mu_1 \neq \mu_2$ (the population means are not equal)

Step 2: Choose the Significance Level

Alpha = 0.05.

Step 3: Select the Test

Because we have two independent samples with unknown population standard deviations and we are assuming equal variances, the two-sample t-test with pooled variance is appropriate.

Step 4: Calculate the Test Statistic

First, calculate the pooled variance:
$s^2p = [(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2] / (n_1 + n_2 - 2)$
$s^2p = [(11)(4.8^2) + (9)(5.3^2)] / (12 + 10 - 2)$
$s^2p = [(11)(23.04) + (9)(28.09)] / 20$
$s^2p = (253.44 + 252.81) / 20$
$s^2p = 506.25 / 20$
$s^2p = 25.3125$

Now calculate the standard error:
$SE = \sqrt{[s^2p(1/n_1 + 1/n_2)]} = \sqrt{[25.3125(1/12 + 1/10)]} = \sqrt{[25.3125(0.08333 + 0.10)]} = \sqrt{[25.3125(0.18333)]} = \sqrt{4.6406} \approx 2.1542$

Finally, the t-statistic:
$t = (\bar{x}_1 - \bar{x}_2) / SE = (85.2 - 80.1) / 2.1542 = 5.1 / 2.1542 \approx 2.3675$

Step 5: Determine the Critical Value

Degrees of freedom = $n_1 + n_2 - 2 = 20$. For a two-tailed test at alpha = 0.05, the critical t-value is approximately ±2.086.

Step 6: Apply the Decision Rule

The calculated t-statistic of 2.3675 exceeds 2.086, so it falls in the critical region. We reject the null hypothesis.

Step 7: Conclusion

There is a statistically significant difference in the mean productivity scores between the two training programs. Program A appears to yield higher productivity scores than Program B.

This second worked solution demonstrates how **testing statistical hypotheses worked solutions** often involve multiple calculations, yet the essential logic remains identical to the simpler one-sample case.

WORKED SOLUTION 3: CHI-SQUARE TEST FOR INDEPENDENCE

Scenario

A market researcher wants to determine whether customer satisfaction level (Satisfied, Neutral, Dissatisfied) is independent of the customer's region (North, South, East, West). A sample of 200 customers yields the following observed frequencies:

	Satisfied	Neutral	Dissatisfied
North	30	10	10
South	25	15	10
East	20	20	10
West	15	15	20

Test at the 0.05 significance level whether satisfaction and region are independent.

Step 1: State the Hypotheses

H_0: Customer satisfaction and region are independent.
H_1: Customer satisfaction and region are not independent.

Step 2: Choose the Significance Level